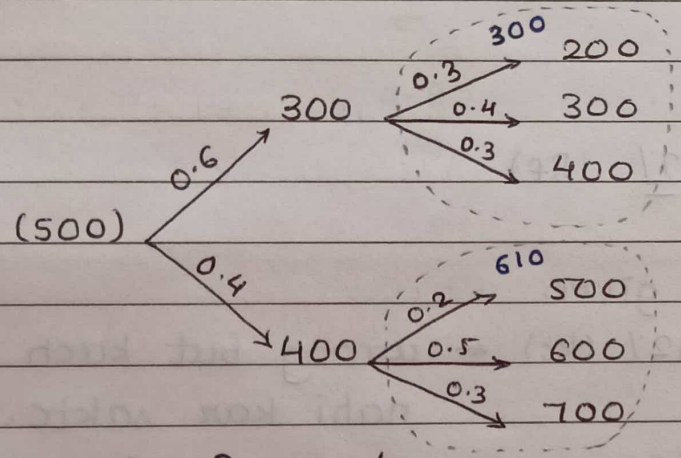
Eg: Consider a risky 2 years project :



Path	Joint Prob.	NPV
1	0.18	-39.1
2	0.24	49.9
3	0.18	138.9
4	0.08	322.2
5	0.20	411.2
6	0.12	500.2

kc = 14%, Rf = 6%

Calculate mean, σ & C.V. of NPV

The firm's existing project have a C.V. which lies in the range of 90% to 110%. Hence firm uses following guidelines for RADR -

Case 1 - C.V. of project < 90%, deduct 2% from kc.

Case 2 - C.V. of project > 110%, add 2% to kc.

Case 3 - C.V. of project falls blw 90% & 110%, use kc.

→ NPV of Path - (using discount rate = Rf = 6%)

1) $300 \times 0.943 + 200 \times 0.89 - 500 = -39.1$
2) $300 \times 0.943 + 300 \times 0.89 - 500 = 49.9$
3) $300 \times 0.943 + 400 \times 0.89 - 500 = 138.9$
4) $400 \times 0.943 + 500 \times 0.89 - 500 = 322.2$
5) $400 \times 0.943 + 600 \times 0.89 - 500 = 411.2$
6) $400 \times 0.943 + 700 \times 0.89 - 500 = 500.2$

Refer soln given by sir for further answer.

Q.] Ok sir, we understood from above eg. that if CF distribution is given, we can apply path approach to generate probability distribution of NPV.
What if we have a 5 years project with 3 CF options per year. How many paths ?

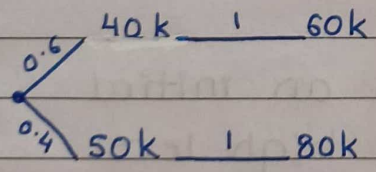
→ Total no. of paths = $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$ paths

BAAP RE !!!!! Is there any shortcut ? YES #
What it is ? Imp.
Why it is ? Not relevant → Derived from MPT.
↳ Modern Portfolio Theory .... separate vdo provided

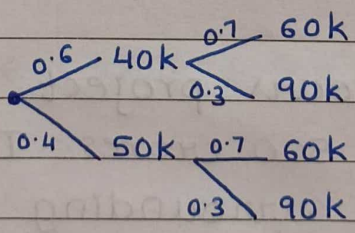
A project can be looked upon as a portfolio of CFs of different years. Hence, σ_{NPV} depends on -

- σ of CF of each year.
- 'r' i.e. correlation blw CFs of different years.

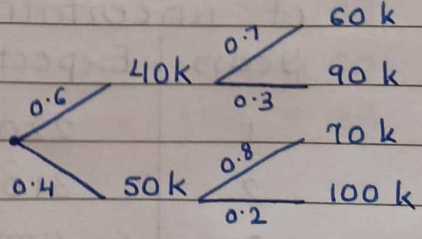
Case 1
Perfectly Correlated CFs
$r = 1$
↓
One to one correspondence



Case 2
Uncorrelated or Independent CFs
$r = 0$
↓
CFs of each year are independent



Case 3
Moderately correlated CFs
↓
CFs of 2nd year depend upon 1st but the dependence is not one to one



Last class eg.
↳ moderate correl^n.

Shortcut is only possible in Case 1 : Perfectly correlated CFs → "HERTZ" model & Case 2 : Uncorrelated CFs → "HILLIER's" model.

Note : Case 3 : Moderately correlated CFs - no shortcut
↳ u have to use path approach .... no option.
Also, do remember that path approach is universal & can also be used in Case 1 & Case 2.
Mind it, in Case 2 - no. of paths can become unbearable .... u remember $3^5 = 243$ paths.

⟹ "HERTZ" Model :-

σ of NPV = P.V. of σ of CFs.

$$= \frac{\sigma_1}{(1+r_f)} + \frac{\sigma_2}{(1+r_f)^2} + \dots\dots + \frac{\sigma_n}{(1+r_f)^n}$$

Irrespective of correlation b/w CFs, expected NPV shall always be calculated in usual manner.

⟹ "HILLIER's" Model :-

- Variance ko tanna hai ..... mtlb disc. krna hai ....
- Double period tanna hai .....
- Last me square root lena hai .....

$$\sigma \text{ of NPV} = \sqrt{\frac{\sigma_1^2}{(1+r_f)^2} + \frac{\sigma_2^2}{(1+r_f)^4} + \dots\dots + \frac{\sigma_n^2}{(1+r_f)^{2n}}}$$

Eg : Consider a 3 years project. It has an initial investment of 500 crores. There is high level of uncertainty surrounding the CFs.

Years	Expected CF	σ of CF
1	220	60
2	310	75
3	450	90

Calculate expected NPV of the project.
Also calculate σ of NPV if -

• k_c : minimum rate of return required by Long Term sources of finance.

Case 1 : CFs are perfectly correlated.
Case 2 : CFs are uncorrelated.
$k_c = 18\%$, $R_f = 7\%$

→ Calculation of expected NPV -

Years	Expected CF	Df @ 7%	Present Value
1	220	0.935	205.70
2	310	0.873	270.63
3	450	0.816	367.20
			843.53
			(500.00) : I
			343.53 : NPV exp^{td}

→ Calculation of σ of NPV -

• Case 1 : Hertz model :

Years	σ of CF	Df @ 7%	Present Value
1	60	0.935	56.10
2	75	0.873	65.48
3	90	0.816	73.44
			195.02 = σ of NPV

• Case 2 : Hillier's Model :

Years (t)	σ_t^2 of CF	Df @ 7% for '2t' years	Present Value
1	3600	0.873	3142.80
2	5625	0.763	4291.88
3	8100	0.666	5394.60
			12829.28

σ^2 of NPV = 12829.28 crores
σ of NPV = 113.27 crores

Imp :-

Suppose we have a 3 year project & CF distribut^n for each year is given

Year 1		Year 2		Year 3	
CF	Probability	CF	Probability	CF	Probability

& I given & Rf given

Obviously, -

Step 1 : We will calculate σ & mean of CF of each year & get data of previous sum.

Step 2 : We will calculate expected NPV in the usual manner.

Step 3 : If the problem is silent regarding correlation b/w the CFs, we must assume that the CFs are Independent & apply Hillier's model.

Q.6 pg

① Step 1 : Calculate of kc & Rf :-

$D/E = 1.5 \Rightarrow w_d = \frac{1.5}{2.5} = 0.6 \quad \therefore w_e = 0.4$

$k_e = 20\%$, $t = 30\%$

$k_d = 14 \times 0.7 = 9.8\%$

$\therefore k_c = w_d k_d + w_e k_e$

$= 0.6 \times 9.8 + 0.4 \times 20$

$k_c = 13.88\%$

$R_f = \frac{1.1388}{1.05} - 1 = 8.46\% \qquad (RP = 5\%)$

② step 2 : Calculation of mean & σ^2 of CFs of different years :-

Year 1				Year 2				Year 3			
CF(x)	P	Px	$P(x-\bar{x})^2$	CF(x)	P	Px	$P(x-\bar{x})^2$	CF(x)	P	Px	$P(x-\bar{x})^2$
10	0.3	3	30	30	0.4	12	0.4	40	0.15	6	0.084
25	0.6	15	15	35	0.2	7	7.2	45	0.5	22.5	9.031
20	0.1	2	0	25	0.4	10	6.4	35	0.35	12.25	11.572
		$\bar{x}$ =	σ_1^2 =			$\bar{x}$ =	σ_2^2 =			$\bar{x}$ =	σ_3^2 =
		20	45			29	14			40.75	20.69

③ Calculation of Expected NPV :-

Years	Expected CF	Df @ 8.46%	Present Value
1	20	0.922	18.44
2	29	0.850	24.65
3	40.75	0.784	31.95
			75.04
		(-) I. Investmt	(60.00)
		Exp. NPV	15.04 lakh

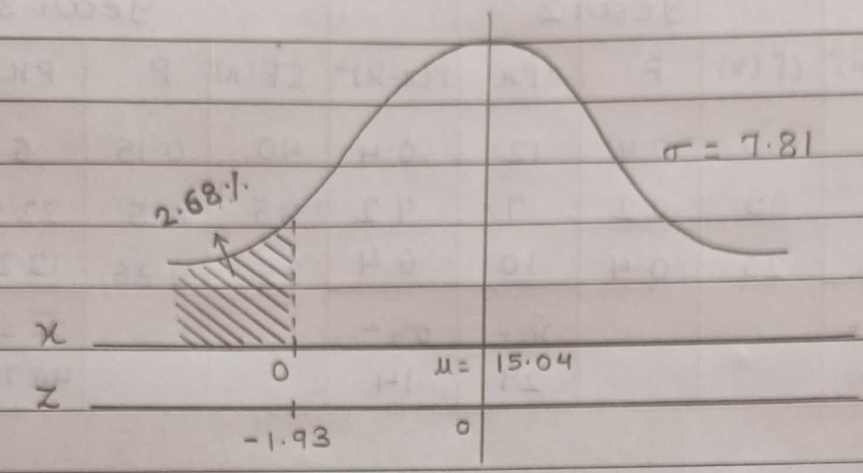
④ Step 4 : Calculation of σ of NPV - Hilliers model

Years (t)	σ^2t of CF	Df @ 8.46% for 2t years	Present Value
1	45	0.850	38.25
2	14	0.723	10.12
3	20.69	0.614	12.70
			61.07

σ^2 of NPV = 61.07 lakhs

σ of NPV = 7.81 lakhs

⑤ Step 5 : Normal Distribution



$$z = \frac{x-\mu}{\sigma} = \frac{0-15.04}{7.81} = -1.93$$

1.93 →

1.90 → 0.0287
1.95 → 0.0256

method 1 = 0.0287 − 0.0186 = 0.0268
i.e. 2.68%.

method 2 = $\frac{0.0287+0.0256}{2}$ = 0.027
i.e. 2.7%.

So, prob. of negative NPV = 2.68%.
which lies within the tolerance
level of 10%. - So, the project is accepted.